****

Meeting

Progressi Recenti in Geometria Reale e Complessa - X

Levico Terme (Trento) – 8-11 ottobre 2016

****

Responsabili Scientifici:

Claudio Arezzo (ICTP Trieste)
Filippo Bracci (Roma II)
Paolo de Bartolomeis (Firenze)

Web-page: http://www.science.unitn.it/cirm/GeometriaReale2016.html

Sponsor

- *FBK-CIRM (Centro Internazionale per la Ricerca Matematica)*
- *INdAM-GNSAGA*
- *HEVO: Holomorphic Evolution Equations. ERC Starting Grant n. 277691*

BOOK OF ABSTRACTS /

SENIOR TALKS

Some open problems in Complex and Symplectic Geometry: three vignettes

Daniel Burns, Jr.

University of Michigan (USA)

Supported in part by NSF grant DMS-1105586.

We discuss, as time permits, three areas where there are open problems at the intersection of complex and symplectic geometry. The first is in toric geometry, where an old problem is the understanding of Delzant theory – the determination of the complex geometry of a space with toric Hamiltonian structure from its moment polytope – to the case of singular systems, typical examples of which are the Gelfand-Cetlin system and integrable geodesic systems. A second is the extension of the DeConcini-Procesi theory of Wonderful Compactifications to reductive complex affine homogeneous spaces which are not necessarily symmetric. This question arises, among other instances, in understanding the behavior at infinity of Grauert tube complexifications of real manifolds. Finally, the third area is related to quantum mechanical interpretation of symplectic geometry. This includes questions such as the exact Bohrr-Sommerfeld conditions for integrable systems and the quantization of non-Hermitian Hamiltonians. This latter instance is related to Donaldson's old proposal concerning the existence of constant scalar curvature Kähler metrics. The solution of that problem leads to questions about the quantum mechanical interpretation of the new stability conditions discovered in Kähler geometry.

Problems in Several Complex Variables

John Erik Fornaess

NTNU Trondheim (Norway)

I will give some general background about the theory of several complex variables and problems that arise. The starting point will be the classical Levi problem. This has led to much development and with many open problems.

Configurations of points in complex projective space

Simon Salamon

King's College London (UK)

I shall present the so-called SIC-POVM problem concerning the existence of $(n+1)^2$ mutually equidistant points in complex projective space CP^n, which relates to embedding CP^n (with its Fubini-Study metric) as an adjoint orbit in $su(n+1)$. I shall sketch a proof that such configurations of 9 points in CP^2 must be standard, using concepts from toric geometry. This is joint work with Lane Hughston.

JUNIOR TALKS

Log-biharmonicity and a Jensen formula in the space of quaternions

Amedeo Altavilla

Università Politecnica delle Marche

Given a complex meromorphic function, it is well defined its Riesz measure in terms of the laplacian of the logarithm of its modulus. Moreover, related to this tool, it is possible to prove the celebrated Jensen formula. In this talk, using among the other things the fundamental solution for the bilaplacian, we introduce a possible generalization of these two concepts in the space of quaternions, obtaining a new interesting Riesz measure and a global (i.e.: four dimensional), Jensen formula.

The content of this talk was developed in collaboration with Cinzia Bisi from Ferrara.

A Camacho-Sad-type index theorem for a couple of holomorphic self-maps

Paolo Arcangeli

Università di Roma "La Sapienza"

Let M be a n-dimensional complex manifold and f and g two distinct holomorphic self-maps of M. Suppose that f and g coincide on a globally irreducible compact hyper surface S of M. We show that if one of the two maps is a local biholomorphism around the regular part S' of S and, if needed, S' sits into M in a particular nice way, then it is possible to define a 1-dimensional holomorphic (possibly singular) foliation on S' and a partial holomorphic connection on the normal bundle of S'. As a consequence, one can localize the $n-1$ power of the first Chern class of the line bundle on M canonically induced by S and thus get an index theorem.

Commuting holomorphic self-maps of the unit ball

Leandro Arosio

Università di Roma Tor Vergata

If f and g are two non-elliptic holomorphic self-maps of the unit disc $\mathbb{D} \subset \mathbb{C}$ such that $f \circ g = g \circ f$, it was proved by Cowen that if f is hyperbolic, then g has to be hyperbolic too (in other words, g cannot be parabolic).

Let $\mathbb{B}^q \subset \mathbb{C}^q$ denote the unit ball. In this talk we will address the following question: can a hyperbolic self-map $f\colon \mathbb{B}^q \to \mathbb{B}^q$ commute with a parabolic self-map $g\colon \mathbb{B}^q \to \mathbb{B}^q$? This question can be answered (almost) completely using the theory of canonical semi-models, that is, natural semi-conjugacies of a self-map $f\colon \mathbb{B}^q \to \mathbb{B}^q$ with some automorphism of a possibly lower dimensional ball $\mathbb{B}^k$. This tool is a generalization to higher dimension of the Schroeder, Valiron and Abel equations in the unit disc $\mathbb{D} \subset \mathbb{C}$. We will show that the answer to the question in higher dimension depends on a new dynamical invariant associated to the self-maps f and g.

This is a joint work with F. Bracci.

A landing theorem for hairs and dreadlocks of entire functions with bounded post-singular sets

Anna Miriam Benini

Università di Roma Tor Vergata

The Douady-Hubbard landing theorem for periodic external rays is one of the cornerstones of the successful study of polynomial dynamics. It states that, for a complex polynomial f with bounded postcritical set, every periodic external ray lands at a repelling or parabolic periodic point, and conversely every repelling or parabolic point is the landing point of at least one periodic external ray. We prove an analogue of the theorem for entire functions with bounded postsingular set. If such f additionally has finite order of growth, then our result states precisely that every periodic hair of f lands at a repelling or parabolic point, and again conversely every repelling or parabolic point is the landing point of at least one periodic hair. (Here a *periodic hair* is a curve consisting of escaping points of f that is invariant under an iterate of f.) For general f with bounded postsingular set, but not necessarily of finite order, the role of hairs is taken by more general connected sets of escaping points, which we call *dreadlocks*. This is joint work with Lasse Rempe-Gillen.

Continuity and non density of stability in the elementary Desboves family

Fabrizio Bianchi

Imperial College, London

For a family of rational maps, results by Lyubich, Mané-Sad-Sullivan and DeMarco provide a fairly complete understanding of dynamical stability. In particular, the Hausdorff continuity of the Julia sets is known to be equivalent to the holomorphic dependence with respect to the parameter, and these conditions of stability are dense in any parameter space.

We will describe a family of endomorphisms of $P^2(C)$ (the two-dimensional analogue of rational maps) for which the Julia set depends continuously on the parameter but never holomorphically. This proves both that the two conditions are in general not equivalent in higher dimension, and that stability is not dense.

This is a joint work with Johan Taflin.

Monotonicity formulas for static metrics in presence of cosmological constant

Stefano Borghini

Scuola Normale Superiore Pisa

We describe a new approach to the study of static metrics in general relativity in presence of cosmological constant. These are triples (M, g_0, u), where (M, g_0) is a n-dimensional Riemannian manifold and u is a smooth function, obeying the system

$$u\,\mathrm{Ric}_{g_0} \;=\; \mathrm{D}^2 u \,+\, \frac{2\Lambda}{n-1}\,u\,g_0 \qquad \text{and} \qquad \Delta_{g_0} u \;=\; -\,\frac{2\Lambda}{n-1}\,u\,.$$

Here $\Lambda \neq 0$ is the so called cosmological constant. Using a conformal deformation of the given solutions, we introduce a new formalism that allows us to treat at the same time both the cases of positive and negative cosmological constant. In particular, we identify natural quantities, that are shown to be monotonic along the level set flow of the static potential u. As a consequence, we derive several sharp inequalities whose equality case is characterized in terms of the rotational symmetry of the solutions. Hence, we recover some uniqueness results for the de Sitter and the anti-de Sitter space-time. Finally, taking advantage of a strong formal analogy, we show how these techniques can be adapted to the study of the classical torsion problem.

The space of Kähler metrics on singular varieties

Eleonora Di Nezza

Imperial College, London (UK)

The geometry and topology of the space of Kähler metrics on a compact Kähler manifold is a classical subject, first systematically studied by Calabi in relation with the existence of extremal Kähler metrics. Then, Mabuchi proposed a Riemannian structure on the space of Kähler metrics under which it (formally) becomes a non-positive curved infinite dimensional space. Chen later proved that this is a metric space of non-positive curvature in the sense of Alexandrov and its metric completion was characterized only recently by Darvas.

In this talk we will talk about the extension of such a theory to the setting where the compact Kähler manifold is replaced by a compact singular normal Kähler space.

As one application we give an analytical criterion for the existence of Kähler-Einstein metrics on certain mildly singular Fano varieties, an analogous to a criterion in the smooth case due to Darvas and Rubinstein.

This is based on a joint work with Vincent Guedj.

ALF spaces and collapsing Ricci-flat metrics on the K3 surface

Lorenzo Foscolo

Stony Brook University (USA)

The Kummer construction of Kähler Ricci-flat metrics on the $K3$ surface provides the prototypical example of the formation of orbifold singularities in non-collapsing sequences of Einstein 4-manifolds. Much less is known about the structure of the singularities forming along sequences of collapsing Einstein metrics. I will describe the construction of large families of Ricci-flat metrics on the $K3$ surface collapsing to the quotient of a flat 3-torus by an involution. The collapse occurs with bounded curvature away from finitely many points. The geometry around these points is modelled by ALF gravitational instantons.

Invariant 2nd order PDEs over adjoint varieties

Jan Gutt

INdAM - Politecnico di Torino

From a geometric point of view, a second order partial differential equation is a hypersurface in a natural bundle of Lagrangian Grassmannians over a contact manifold. In the compact complex setting, a rational homogeneous contact manifold is precisely the adjoint variety of some simple Lie group G. Does it admit G-invariant 2nd order PDEs? Are any of them distinguished? I will address these questions reporting on joint work with D. Alekseevsky, G. Manno and G. Moreno.

Analogues of coverings and the fundamental group in geometry of partial differential equations

Sergei Igonin

INdAM - Politecnico Torino

It is known that, using jet bundles, one can regard partial differential equations (PDEs) as geometric objects. Namely, a PDE can be regarded as a manifold with a distribution such that solutions of the PDE correspond to certain integral submanifolds of the distribution. This allows one to study (nonlinear) PDEs by means of methods of differential geometry.

Recall that fundamental groups are an important invariant for topological spaces. In this talk, we introduce an analogue of fundamental groups for PDEs. However, the "fundamental group of a PDE" is not a group, but a certain system of Lie algebras, which we call fundamental Lie algebras. Fundamental Lie algebras are new geometric invariants for PDEs and are closely related to integrability properties of PDEs, where integrability is understood in the sense of soliton theory.

In particular, using fundamental Lie algebras, we obtain necessary conditions for integrability and necessary conditions for existence of Backlund transformations for nonlinear $(1 + 1)$-dimensional evolution PDEs. In the structure of fundamental Lie algebras for integrable $(1 + 1)$-dimensional PDEs, one finds infinite-dimensional subalgebras of Kac-Moody algebras and infinite-dimensional Lie algebras of certain matrix-valued functions on some algebraic curves. To develop this theory, we use a generalization of the Wahlquist-Estabrook prolongation method and the theory of coverings of PDEs invented by A. Vinogradov and I. Krasilshchik. Some results of this talk have been obtained in joint works with G. Manno (Politecnico di Torino).

From toric cusps to toric funnels

Samuele Lancini

Scuola Normale Superiore Pisa

In the setting of Riemann manifolds with constant negative scalar curvature, we discuss about the possibility to replace cusps with funnels by means of a perturbation. This perturbation guarantees the existance of a minimal torus in each funnel. In the two-dimensional case, we get back the classic results about hyperbolic surfaces without reference to the Teichmuller theory.

Towards Fultons conjecture on the ample cone of the moduli space of rational curves with marked points

Paolo Lella

Università di Trento

Abstract Let $\overline{M}_{0,n}$ be the compactified moduli space of stable rational curves with n marked points. A well-known conjecture related to the study of the ample cone of $\overline{M}_{0,n}$ says that if a divisor D intersects non-negatively an explicit class of one-dimensional subvarieties called Faber-curves (or F-curves), then D is linearly equivalent to an effective combination of boundary divisors, i.e divisors corresponding to the boundary components of $\overline{M}_{0,n}$, whose general point parameterizes the union of two rational curves. The conjecture is known to be true for $n \leqslant 7$ and a counterexample to the conjecture was produced for $n \geqslant 12$. During the talk, I will present a new proof of the conjecture for $n = 7$ based on techniques borrowed from convex geometry.

Zoll metrics of revolution on spheres

Simon Lohove

Università di Firenze

A metric g on a manifold M is called a Zoll metric if all of its unit speed geodesics are simple closed and of common lenght l. We denote the class of these metrics by SC_l if we want to fix the periodicity. In this talk, I will present a classification of $SC_{2\pi}$ metrics of revolution on $\mathbb{S}^n$, i.e Zoll metrics of periodicity 2π on the cohomogeneity one manifold with effective group diagram $SO(n-1) \subset SO(n), SO(n) \subset SO(n)$.

Nice embeddings of compact homogeneous CR manifolds

Stefano Marini

Università di Roma Tre

The complex Matsuki dual of the orbit of a real form in a complex flag manifold has a Dolbeault cohomology which is nicely related to the CR cohomology of the compact intersection. In fact, the Mostow fibration yields canonical exhaustions that can be used to apply the Andreotti-Grauert theorem.

A sphere theorem for harmonic potentials

Giovanni Mascellani

Scuola Normale Superiore Pisa

The classical Laplace problem with a Dirichlet boundary condition on an exterior domain of R^n is considered. It is shown that, in presence of appropriate additional conditions, the solution must have rotational symmetry. This "overdetermining" condition is of extremal type, i.e., it corresponds to the equality case of a general geometric inequality, which is proved to hold for solutions on any exterior domain.

In order to prove the result, a conformal transformation of the exterior domain is proposed, giving rise to an equivalent formulation (spherical ansatz) that can be studied with classical techniques of Riemannian geometry. Proving the rotational symmetry of the original solution amounts to prove a sphere theorem for the conformally deformed metric.

This is joint work with L. Mazzieri and S. Borghini and follows related results by V. Agostiniani and L. Mazzieri.

Geometric analysis on stratified spaces

Ilaria Mondello

Université Paris Est Créteil

In this talk we will give a brief introduction to the singular setting of stratified spaces, which generalize the notion of isolated conical singularity. We will present some geometric and analytic tools inspired by classical Riemannian geometry, in order to deduce a lower bound for the first non-zero eigenvalue of the Laplacian under the appropriate lower bound for the Ricci curvature.

Misure di Carleson e operatori di Topelitz in domini strettamente pseudoconvessi limitati

Samuele Mongodi

Università di Pisa

Le misure di Carleson furono introdotte da Carleson per studiare il problema della corona; una misura μ di Carleson per uno spazio di Banach A di funzioni olomorfe se A si immerge con continuit in $L^p(\mu)$. Una delle prime domande che ci si pone e' come caratterizzare le misure di Carleson tramite propriet geometriche del dominio, preferibilmente invarianti per biolomorfismo. Per un generico limitato strettamente pseudoconvesso e funzioni olomorfe L^q, questo e' stato fatto da Abate, Raissy e Saracco, stabilendo un legame tra le misure di Carleson, la trasformata di Berezin e gli operatori di Toeplitz, entrambi questi ultimi definiti in termini del nucleo di Bergman del dominio. Nel mio seminario, presenteró questi risultati e accenneró ad alcune generalizzazioni ed estensioni.

Representations of compact Lie groups with a generalized toric reduction

Francesco Panelli

Università di Firenze

Given an orthogonal representation (G, V) of a compact conected Lie group G on a metric vector space V, it is an interesting problem to understand which propreties of (G, V) can be recovered from the metric properties of the orbit space V/G. While it has been proved that reducibility of (G, V) depends only on the quotient space V/G, a recent result by Claudio Gorodski and Alexander Lytchak asserts that reducibility of the induced representation (G, V) of the identity component G of G does not; moreover, irreducible representations for which this happens have been completely classified and it turns out that they are exactly representations admitting a generalized toric reduction.

After a brief introduction to the topic, in this talk we will focus on reducible representations admitting a generalized toric reduction and, in particular, on those with abstract copolarity 1 and 2, proving some recent and new results towards their description.

Contact twisted cubic structures

Katja Sagerschnig

INdAM - Politecnico Torino

It is a classical concept to study PDEs through their characteristic variety. For instance, Monge-Ampere equations admit a particular simple characteristic variety that we can identify with a line field in a projective space. For other types PDEs one obtains more general curves, such as plane curves or twisted cubics. This observation motivates the study of what we call punctured contact twisted cubic structures. In this talk we discuss the history and geometry of these structures, the relation to the exceptional Lie group G_2, and present preliminary results towards a classification of the considered geometries. The talk is based on joint work (in progress) with G. Manno, G. Moreno and P. Nurowski.

Cohomological aspects on complex and symplectic manifolds

Nicoletta Tardini

Università di Pisa

We discuss quantitative and qualitative cohomological properties on compact complex and symplectic (not necessarily Kähler) manifolds. In particular, we focus on the Bott-Chern and Aeppli cohomologies in these two settings. We will provide quantitative characterizations of the $\partial\overline{\partial}$-lemma on complex manifolds, and of the Hard Lefschetz Condition on symplectic manifolds in terms of the dimensions of these cohomology groups.

These are joint works with Daniele Angella and Adriano Tomassini.

Classification of metrics admitting one projective vector field

Andreas Vollmer

INdAM - Politecnico Torino

In 1882 Sophus Lie posed the problem of classifying 2-dimensional (pseudo-)Riemannian manifolds (M,g) admitting one or several projective vector fields, i.e. vector fields whose local flow sends geodesics to geodesics (viewed as unparametrized curves). For the case when the projective action is locally transitive on M a complete classification has been given in 2007 by Bryant, Manno & Matveev, whereas in the non-transitive case only partial results exist. The talk will present ongoing research on this classification, aiming at a local classification of metrics that admit one projective vector field.
